

Interactively animating crumpling paper

Camille SCHRECK
Univ. Grenoble-Alpes,
CNRS (LJK),
Inria, France

Stefanie HAHMANN
Univ. Grenoble-Alpes,
CNRS (LJK),
Inria, France

Damien ROHMER
Univ. Grenoble-Alpes & CPE
Lyon
CNRS (LJK),
Inria, France

Marie-Paule CANI
Univ. Grenoble-Alpes,
CNRS (LJK),
Inria, France

ABSTRACT

We present the first method in computer graphics to animate sheets of paper at interactive rates while automatically generating a plausible set of sharp features when the sheet is crumpled. Our hybrid, geometric and physical, model is based on a high-level understanding of the physical constraints that act on real sheets of paper, and of their geometric counterparts.

This understanding enables us to use an adaptive mesh carefully representing the main geometric features of paper in terms of singular points and developability.

Categories and Subject Descriptors

I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism

Keywords

Computer Animation, Geometric Modeling, Interactive Deformation, Paper Crumpling, Developable Surface

1. INTRODUCTION

Computer animations of crumpled paper sheets are almost absent from virtual environments although it is a very common material in everyday life. This is due to the extreme difficulty of efficiently capturing the dynamic, non-smooth developable geometry of paper sheets. Indeed the microscopic fibers structure significantly affects the macroscopic behavior and makes it challenging to reproduce. The goal of this work is to achieve the animation of paper crumpling processes at interactive rates. This may greatly simplify the work of artists who need to model this phenomenon. Let us list the macroscopic features that need to be captured in order to crumple a sheet of paper in a visually plausible way:

Length preservation. The assumption can be made that the surface of a sheet of paper preserves lengths with respect to an original 2D pattern. It can be flattened onto a plane without distortion (stretching or compressing). This is a property which is called developability.

Sharp features. Crumpled sheets of paper exhibit singular points, and possibly some sharp creases.

Plastic behavior and shape memory. In real life, the microscopic fibers of paper or more precisely the bonds between them may break upon deformation, causing irreversible damages to the structure.

The modeling of paper has been studied in numerous previous works that can be divided into two main approaches :

Geometric modeling. Bending and creasing virtual paper can be expressed through geometric deformation [7]. An interactive tool is presented in [18] to fold thin sheets of materials. Other methods address interactive origami modeling [15, 9, 14] where the folds are explicitly defined by the user. A geometric way to create a virtual sheet of paper with sharp features from a 3D boundary curves and a 2D pattern is proposed in [11] but cannot be extended to animation.

Physically based modeling. Thin sheet materials are typically modeled in computer graphics using 2D finite elements with a linear elastic behavior. This approach has been used to bend sheets of paper using the thin plate theory [5, 2]. However, this method cannot handle the dynamic formation of singular points and sharp edges. Kang *et al.* [6] propose to model the plastic behavior of the paper using breakable springs, but the approach suffers from visual artifacts as edge-split and singularities can only occur along preexisting coarse triangle edges. Recently Narain *et al.* [10] obtained high-quality visual simulation of paper by combining this inelastic deformation model with an adaptive remeshing method. Yet, this approach is computationally expensive due to the large number of triangles in the folded regions, so it cannot be used in interactive applications.

The key idea of this work is to obtain interactive rates and automatic generation of sharp feature by interleaving standard physically-based simulation steps with procedural generation of a piecewise continuous developable surface.

Our contributions include a hybrid physically-based and geometric model, a realistic modeling of singular points, an optimal adaptive isometric meshing and a validation through comparison with real paper.

2. A NEW HYBRID MODEL FOR PAPER

As already mentioned, a sheet of paper can be considered as a developable surface, which can be defined as a ruled surface with zero Gaussian curvature (product of the principal curvatures) everywhere. As a consequence, if a smooth part of the paper is bent such that one principal curvature is not zero, then the other principal curvature in the direction of the fold is necessarily zero. Thus the surface becomes rigid in the direction of zero-curvature, leading discontinuities to appear instead of having the surface bend if a compression is applied in the orthogonal direction.

The appearance of singularities has also been studied in the mechanical literature: due to the fact that the bending rigidity of thin plates is much smaller than their stretching rigidity, the paper mechanical transformations are favorable to bending [3]. In addition, thin sheets have the specific behavior of concentrating elastic energy when being constrained [17]. When the thickness tends to 0, this energy concentrates into singular points, which are called d-cones [1, 8, 4].

These d-cones denote developable generalized cones defined by a fixed point called the *apex* and a one-parameter family of straight lines (rulings) passing through the apex. Outside of the influence of the d-cones, the surface is smooth and exhibits a low distribution of elastic energy [13]: it can be deformed according to standard linear models for continuous mechanics laws.

We choose thus to have the motion of the surface guided by an usual physic simulation based on continuous mechanics. The surface model used to represent the shape of a sheet of paper must be highly deformable, as sharp features may appear anywhere during deformation. This usually requires to use dense meshes, which provide a large number of degrees of freedom and reduce visual artifacts due to badly oriented edges. Instead we choose to dynamically adapt the mesh to the surface using a dedicated geometric model in order to have the necessary degrees of freedom for surface deformation while keeping a coarse mesh. This reduces most of the time usually spent in physically-based simulation.

The observations made above lead us to use the following model to get an optimized mesh (represented in Figure 1). The surface of the paper is approximated by a set of curved regions (in magenta), represented by generalized cones, and planar regions (in green). The mesh used by the physically-based simulation is composed of the triangles and quadrangles formed by the rulings of generalized cones and of a very coarse triangulation of the planar regions. In this way, the curved regions are rigid along the direction of the rulings, as for real paper, while planar regions are still able to bend in any direction.

The sharp creases of the paper are modeled by singular points (red dots) which can only be apices of d-cones. They are generated in the regions where conflicts between two bending directions arise. Their positions are computed using a probability law in the conflict's region. As the apparition of sharp points is a plastic event, we record the position of these vertices on the 2D-pattern.

We interleave the updating of the mesh with the simulation steps in order to obtain a simplified, yet visually realistic paper behavior. The optimized mesh stays coarse such that the animation can be achieved in interactive time. Moreover, during the whole animation process, we are able to maintain an approximative isometric mapping between the

paper surface S and its pattern $\bar{S}$ defined in the 2D parameter domain.

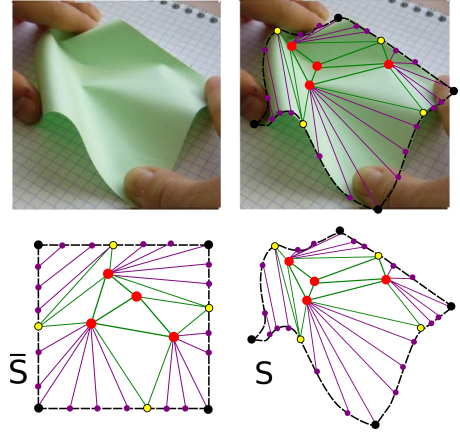


Figure 1: Photograph of a sheet of paper bent in several directions (top left). Our geometric surface representation (pattern on the bottom left, and 3D surface on the bottom right) is divided into curved regions (magenta) and planar regions (green). Singular vertices (red dots) are apices of generalized cones.

We propose the following algorithm, a more detailed description can be found in [12].

Initialization. The input is a developable surface S and its isometric mapping to a 2D pattern $\bar{S}$. Additionally, some *handles* (black dots in Figure 1), i.e. a finite set of points that the user can manipulate, are defined. They act as hard constraints and are used to govern the deformation.

Animation loop. (see Figure 2)

- **Step 1: Elastic deformation.** A state-of-the-art physically-based simulation is applied for efficiently deforming the surface mesh in a plausible way but without taking care of paper plasticity at this stage. Isometry to $\bar{S}$, and therefore developability, may thus be lost.
- **Step 2: Modeling bending and crumpling.** The connectivity of the mesh is modified to update the position of the curved regions and the flat regions. Notably the conflicts that may appear between two regions bent in different directions are solved by introducing new singular vertices.
- **Step 3: Developable**

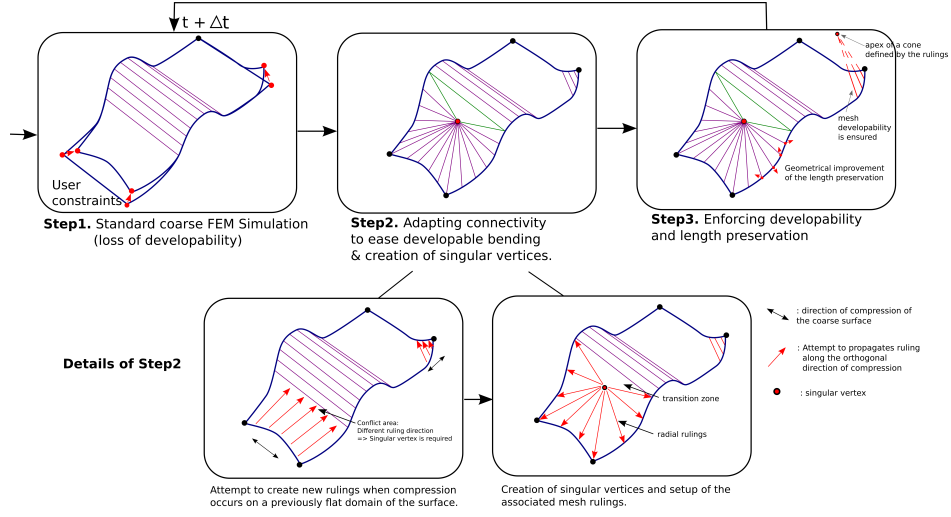


Figure 2: Overview of the algorithm

the two opposite sides of an smooth flat square sheet of paper (10cm×10cm) closer to each other by a factor of 10%. Then we applied an orthogonal constraint on one of the borders until a singular point appeared (see Figure 3-left). We repeated the experiment 20 times with other sheets of the same thickness and then 20 times using our method. The resulting distributions of the position of the first singular point are depicted in Figure 3 (center and right), for two different types of paper. Our results show a good accordance between real measurements and our virtual model.

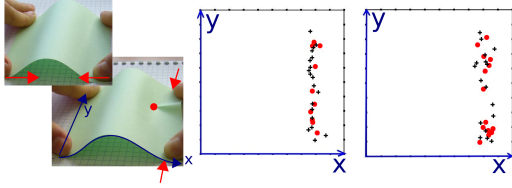


Figure 3: Comparison of the positions of the first appearing singular vertex. *Left: deformations applied to the sheet of paper. Right: the measurements on real paper (black crosses) and the positions generated by our method (red dots) for a 70 g/m² paper (center) and a 90 g/m² paper (right). We use different standard deviation for computing the position of the singular points in order to take into account the different type of paper.*

Visual comparison between real paper and our results are also provided in Figure 4. The top row illustrates the deformation of a smooth paper strip continuously twisted into a Moebius strip. The second row compares the behavior of a sheet of paper after a singular vertex is inserted: we get the same effect of the sheet straightening when the constrained points at the back are unloaded.

Comparison with other methods. Figure 5 compares our method with a model using a standard finite element method applied to a fine mesh of fixed connectivity, and with one using adaptive remeshing from Narain *et al.* [10]. All three methods use the same physical FEM solver, and are able to obtain comparable visual results. Our method requires approximately ten times less triangles than the adap-



Figure 4: Comparing our results with real paper deformations.

tive remeshing method in Narain *et al.* and twenty times less than the standard FEM approach with fixed connectivity for similar visual result, leading to far better time rates (see Table 1).

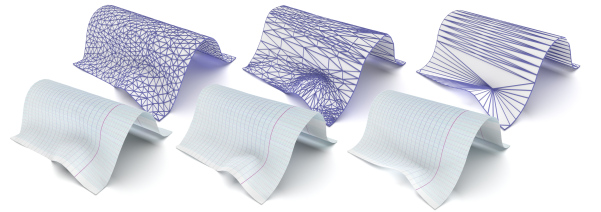


Figure 5: Comparison with other methods. *The Standard FEM (left) and Narain *et al.* [10] method (middle) required a very dense mesh in order to get a result of similar visual quality as our method (right).*

Other results. Table 2 provides the average length distortion over all the animation frames and the computational times for the different examples. Figure 6 gathers other re-

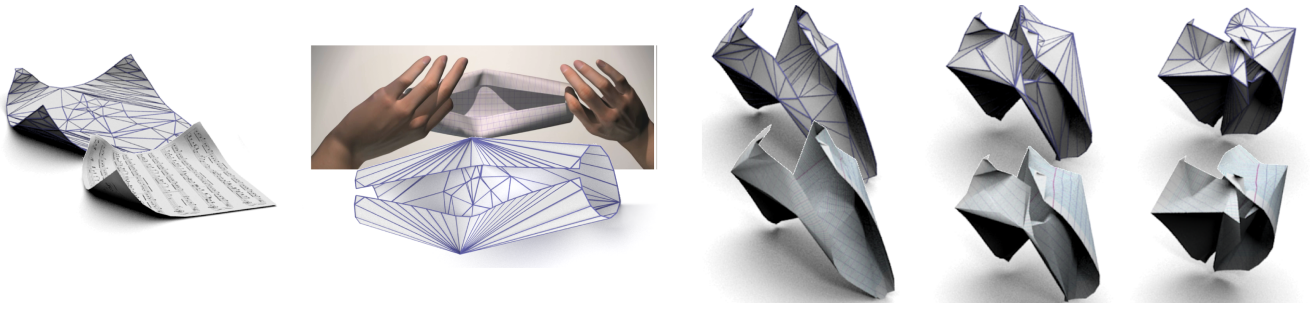


Figure 6: Some other results.

Table 1: Average computation times.

	Our method	standard FEM	Narain <i>et al.</i>
Remeshing time (ms)	1	-	450
Simulation time (ms)	100	14,700	4,640
Nb. of triangles	89	1668	788

Computed for the example presented in Fig. 5.

Table 2: Measures of error in length and Computational time analysis.

Models	c_m	N_t	t_{sim} (ms)	t_{geom} (ms)
Fig. 4 (top)	0.17	117	294	3.3
Fig. 4 (bottom)	0.19	87	60	1.4
Fig. 6 (left)	1.47	156	247	2.8

c_m is the mean percentage of compression over all the model edges and N_t is the average number of triangles of the mesh through the animation. t_{simu} and t_{geom} are the respective times spent in average in the simulation steps versus in the geometrical steps.

sults of our method.

4. CONCLUSION

We get a variety of results that qualitatively look as real paper, indicating that our method has reached most of its goals.

We would like to extend our work to have creased curves and not only singular points as sharp features, as shown in mechanical studies. Also, although it is much faster than state-of-the-art methods, our prototype currently runs at 1 to 10 fps. As future work, we would like to improve this method to reach real-time performances.

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